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General quantum phase estimation and calibration of a timepiece in a quantum dot system

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Abstract

We present a physical scheme for implementing quantum phase estimation via weakly coupled double-quantum-dot molecules embedded in a microcavity. During the same process of implementation, we can also realize the calibration of a timepiece based on the estimated phase. We use the electron–hole pair states in coupled double-quantum-dot molecules to encode quantum information, where the requirement that two quantum dots are exactly identical is not necessary. Our idea can also be generalized to other systems, such as atomic, trapped ion and linear optics systems.

1. Introduction

Relative phase plays an important role in quantum information. The encoding of information into the relative phase of quantum systems has been extensively used in quantum cryptographic [1], quantum cloning [2], geometric quantum computation [3] and so on. Phase estimation based on the discrete quantum Fourier transform (QFT) is a comparatively good method for resolving some phase problems. The phase estimation is a procedure for measuring a certain unknown phase with high precision, which is also the key ingredient for resolving some complex quantum algorithms [4–6], e.g. the factoring problem and the order-finding problem. Therefore quantum phase estimation is a very important tool in quantum communication and quantum computation.

In order to estimate an unknown phase ϕ ($\phi \in (0, 2\pi]$), we must use an oracle in the process because the phase estimation procedure is not a complete quantum algorithm in its own right. At the same time, the generation of a state $|u\rangle$ with an eigenvalue $e^{i\phi}$ is necessary. In addition, we should also find a unitary transformation U, which satisfies

$$U|u\rangle = \exp(i\phi)|u\rangle.$$

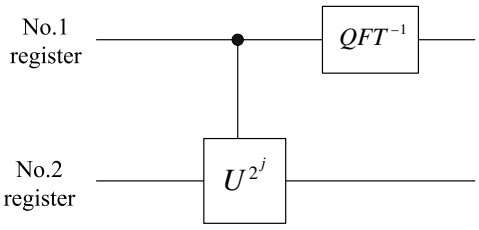

Figure 1. The sketch map for the procedure of phase estimation.

Controlled unitary transformations C–U^{2^j} ($j \in \mathbb{N}^+$) will be performed in the process of the oracle [7]. The main elements of quantum phase estimation are the oracle transformation and an inverse QFT, the sketch of which is shown in figure 1. The No.1 register contains m qubits initially prepared in the state $|0\rangle^{\otimes m}$ while the eigenstate $|u\rangle$ was encoded into the No.2 register. The detailed process of phase estimation can be described as follows: firstly, perform a Hadamard gate operation on each of the m qubits in the No.1 register. Secondly, apply appropriate C–U^{2^j} operations to the whole system with the m qubits in the No.1 register used as controlled bits while $|u\rangle$ is used as a target bit. Then apply an inverse QFT on the qubits in No.1 register. Finally, measure the output of No.1 register. According to the measurement result, we can estimate the unknown phase $\phi \simeq \widetilde{\phi}$. The successful probability and the number of digits of accuracy we wish to have in the estimation depend on m.

Recently, many researches on phase estimation have been presented including the lower bound for phase estimation [8], optimal phase estimation for qubits in mixed states [9], optimal phase measurements with pure Gaussian states [10] and optimal quantum circuits for general phase estimation [11]. However, the implementation of quantum phase estimation in physical systems is not a easy task since an unknown phase is involved in the procedure. To overcome this difficulty, we can introduce a fungible magnitude T into the procedure of phase estimation. A solid-state system would be the most promising candidate for quantum computer use considered by scientists. Recently one of the solid-state systems—a quantum dot system—has attracted much attention because of its intrinsic properties. In the realm of the quantum dot, electronic charge states [12, 13], single-electron spin states [14, 15], the spin singlet state and triple states of double electrons [16, 17] can all be used as qubits to encode quantum information. In particular, schemes combining cavity technology become very useful for quantum information processing because the cavity mode can be used as a data bus for long-distance information transfer or long-distance fast coupling between two arbitrary qubits. In comparison with those on other transmission media, the parallel operations on two arbitrary different qubits can be more easily realized by using cavity technology. Moreover the spatial separation of electronic charge states can enhance quantum coherence [18]. Therefore we investigate the implementation of quantum phase estimation via the interaction between weakly coupled double-quantum-dot molecules and the microcavity in this paper. Because we introduce the new fungible magnitude, we can calculate time T in terms of the final measurement result, which corresponds to the phase $\widetilde{\phi}$. Then we can calculate the error of time comparing with an ideal clock. If the error is within the range of the precision η ($\eta = \widetilde{\phi}/\phi \times 100\%$) of the phase estimation, the error will be neglected; otherwise, the frequency of the time should be regulated.

2. Implementation of phase estimation and calibration of a timepiece

In this section, we discuss a scenario for implementing quantum phase estimation in detail. During this process, we also can check a timepiece: whether it is precise or not, by comparing with an ideal timepiece.

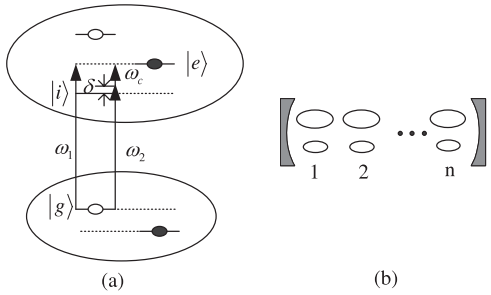

Figure 2. (a) Configuration of a weakly coupled double-quantum-dot molecule. Two ellipses present two arbitrary quantum dots, the ground state denoted by $|g\rangle$ is used for the qubit logic state $|\widetilde{0}\rangle$, the excited state $|e\rangle$ for logic state $|\widetilde{1}\rangle$, and $|i\rangle$ is an intermediate state. ω_1 and ω_2 are two frequencies of pulse lasers, and ω_c is the frequency of the cavity photon. (b) n quantum dot molecules are embedded in a microcavity. Assume that the distance between two neighboring quantum dot molecules is large enough for neglecting Coulomb correlations.

2.1. Interaction between weakly coupled double-quantum-dot molecules with laser fields and the microcavity

In our scheme, we use electronic charge (electron–hole pair) states to store information; the configuration diagram of a qubit is shown in figure 2(a). The states $|g\rangle$, $|e\rangle$ and $|i\rangle$ result from the conduction and valence band states of the two individual quantum dots with different sizes [12]. All of the quantum dot molecules are embedded in a microcavity. Assume that there is no intermediate state between the two lowest conduction band states and the highest valence band state. If we use a pulse laser on a coupled double-quantum-dot molecule with frequency ω_1, the Rabi transition $|g\rangle \leftrightarrow |e\rangle$ can be governed by the following interaction Hamiltonian ($\hbar = 1$) [12]:

$$H_{\mathrm{I}} = \Omega_1(|e\rangle\langle g|\mathrm{e}^{\mathrm{i}\phi_1} + |g\rangle\langle e|\mathrm{e}^{-\mathrm{i}\phi_1}), \tag{1}$$

where Ω_1 is the Rabi frequency, and ϕ_1 is the laser phase. We can obtain the evolution after a duration time t

$$|g\rangle \rightarrow -\mathrm{i}\mathrm{e}^{-\mathrm{i}\phi_1}\sin(\Omega_1 t)|g\rangle + \cos(\Omega_1 t)|e\rangle, \tag{2a}$$

$$|e\rangle \rightarrow \cos(\Omega_1 t)|g\rangle - \mathrm{i}\mathrm{e}^{\mathrm{i}\phi_1}\sin(\Omega_1 t)|e\rangle, \tag{2b}$$

from which we can realize arbitrary single-qubit transformations by adjusting Ω_1, t and ϕ_1.

If we switch on a pulse laser with frequency $\omega_2 = E_{\mathrm{e}} - E_{\mathrm{g}} - \omega_{\mathrm{c}}$, then the ω_2 laser photon and the ω_{c} cavity photon will participate in a resonant transition $|g\rangle \leftrightarrow |e\rangle$; the interaction Hamiltonian can be written as [12]

$$H_{\mathrm{II}} = \Omega_{\mathrm{eff}}(|e\rangle\langle g|a\mathrm{e}^{\mathrm{i}\phi_2} + |g\rangle\langle e|a^{\dagger}\mathrm{e}^{-\mathrm{i}\phi_2}), \tag{3}$$

where $\Omega_{\mathrm{eff}} = \Omega_{\mathrm{c}}\Omega_2/\delta$; $\delta = \omega_2 - (E_i - E_{\mathrm{g}})$ is the detuning between the laser frequency and the energy of transition from $|g\rangle$ to $|e\rangle$ during this transition; Ω_2 and Ω_{c} are the strengths of coupling of $|g\rangle \leftrightarrow |i\rangle$ and $|i\rangle \leftrightarrow |e\rangle$, respectively. There is no occupation of the intermediate state $|i\rangle$ because of the existing large detuning δ. We can obtain the time evolution corresponding to H_{II} as

$$|g\rangle|0\rangle \rightarrow |g\rangle|0\rangle, \tag{4a}$$

$$|g\rangle|1\rangle \rightarrow \cos(\Omega_{\mathrm{eff}}t)|g\rangle|1\rangle - \mathrm{i}\mathrm{e}^{\mathrm{i}\phi_2}\sin(\Omega_{\mathrm{eff}}t)|e\rangle|0\rangle, \tag{4b}$$

$$|e\rangle|0\rangle \rightarrow \cos(\Omega_{\text{eff}}t)|e\rangle|0\rangle - \mathrm{i}\mathrm{e}^{-\mathrm{i}\phi_2}\sin(\Omega_{\text{eff}}t)|g\rangle|1\rangle, \tag{4c}$$

$$|e\rangle|1\rangle \rightarrow |e\rangle|1\rangle. \tag{4d}$$

This process of evolution is the essential ingredient for realizing arbitrary two-qubit operations in this system, such as the controlled-not gate [12] and the controlled phase flip, where the photonic state ($|0\rangle$ or $|1\rangle$) is used to mediate the coupling between two arbitrary qubits.

2.2. Implementation of general quantum phase estimation

To implement quantum phase estimation, we prepare two clocks (clock 1 is a precise one; the frequency of clock 2 is unknown but its scale is well proportioned), a vacuum microcavity mode state $|0\rangle$, and $m+1$ coupled double-quantum-dot molecules without excess electrons in their conduction bands, where m molecules are all initialized in $|g\rangle^{\otimes m} = |\widetilde{0}\rangle^{\otimes m}$, and the $(m+1)$ th molecule is in $|e\rangle_{m+1} = |\widetilde{1}\rangle_{m+1}$. The detailed scenario for implementing general quantum phase estimation can be described as the following three steps:

(I) Firstly, perform a Hadamard gate operation on each of the quantum dot molecules, from molecule 1 to molecule m, respectively, which can be realized by an interaction such as that in equation (1). Here we choose $\phi_1 = 2k\pi + \pi/2$ and $\Omega_1 t/\hbar = 2n\pi + \pi/4$ $(k, n \in \mathbb{N})$. The time t is detected by clock 1. Then we should perform a controlled phase C–U gate on quantum dot molecules m and $m+1$ (molecule m is used as control bit while molecule $m+1$ is used as the target bit) by an interaction such as that in equation (3). However, molecule $m+1$ remains in the state $|\widetilde{1}\rangle$ at all times, so we only need to operate a single-qubit ϕ phase gate on molecule m to achieve the above task (the C–U gate) by using an interaction such as that in equation (1) by choosing $\Omega_1 t/\hbar = 2n\pi + \pi/2$ and $\phi_1 = \phi + \pi/2$ (the ϕ is unknown and can be controlled by an unknown length l of an electro-optic crystal, so it also can be controlled by the time T of going through the electro-optic crystal). The time T is detected by clock 2. Similarly, we perform 2 times ϕ phase transformations on molecule $m-1$, perform 4 times ϕ phase transformations on molecule $m-2, \cdots$, and perform 2^{m-1} times ϕ phase transformations on molecule 1. After that, the state of the total system becomes

$$\begin{aligned}|\psi\rangle &= \frac{1}{2^{m/2}}(|\widetilde{0}\rangle_1 + \mathrm{e}^{\mathrm{i}2^{m-1}\phi}|\widetilde{1}\rangle_1)(|\widetilde{0}\rangle_2 + \mathrm{e}^{\mathrm{i}2^{m-2}\phi}|\widetilde{1}\rangle_2)\cdots(|\widetilde{0}\rangle_m + \mathrm{e}^{\mathrm{i}2^0\phi}|\widetilde{1}\rangle_m)|\widetilde{1}\rangle_{m+1} \\ &= \frac{1}{2^{m/2}}\sum_{k=0}^{2^m-1}\mathrm{e}^{\mathrm{i}\phi k}|k\rangle. \end{aligned} \tag{5}$$

(II) Setting $\phi = 2\pi\varphi$, assume that φ can be expressed exactly in m qubits, so $\varphi = 0.\varphi_1\cdots\varphi_m$ ($\varphi_i = 0$ or 1), where $0.\varphi_1\cdots\varphi_m = \varphi_1/2 + \varphi_2/4 + \cdots + \varphi_m/2^m$. The state of the quantum dot molecules from molecule 1 to molecule m can be rewritten as

$$|\psi\rangle = \frac{1}{2^{m/2}}(|\widetilde{0}\rangle_1 + \mathrm{e}^{2\pi\mathrm{i}0\cdot\varphi_m}|\widetilde{1}\rangle_1)(|\widetilde{0}\rangle_2 + \mathrm{e}^{2\pi\mathrm{i}0\cdot\varphi_{m-1}\varphi_m}|\widetilde{1}\rangle_2)\cdots(|\widetilde{0}\rangle_m + \mathrm{e}^{2\pi\mathrm{i}0\cdot\varphi_1\cdots\varphi_m}|\widetilde{1}\rangle_m). \tag{6}$$

Then perform an inverse QFT on the No.1 register; the detailed process can be described as follows. (1) We perform a Hadamard transform on quantum dot molecule 1; the state of quantum dot molecule 1 becomes $|\varphi_m\rangle$. (2) We perform a series of operations on quantum dot molecules 1 and 2: we perform a single-qubit $-\theta$ ($\theta = \pi/4$) phase gate operation on quantum dot molecule 1, a controlled-not gate operation on quantum dot molecules 1 and 2 (molecule 1 is used as a control bit while molecule 2 is used as a target bit), a single-qubit θ phase gate operation on molecule 2, a controlled-not gate operation on quantum dot molecules 1 and 2 again, and a single-qubit $-\theta$ phase gate operation on molecule 2. These operations on molecules 1 and 2 can be expressed as a total transformation

$U_{12} = U_2(-\theta)U_{12}(\text{cnot})U_2(\theta)U_{12}(\text{cnot})U_1(-\theta)$. Then we perform a Hadamard transform on quantum dot molecule 2. The state of quantum dot molecule 2 becomes $|\varphi_{m-1}\rangle$. (3) Similarly, we apply the transformation U_{13} on molecules 1 and 3 with $\theta = \pi/8$, and the transformation U_{23} on molecules 2 and 3 with $\theta = \pi/4$ as in step (2). Then we perform a Hadamard transform on quantum dot molecule 3. The state of quantum dot molecule 2 becomes $|\varphi_{m-2}\rangle$; $\cdots$. (m) We apply the transformation U_{1m} on molecules 1 and m with $\theta = \pi/2^{m-2}$, the transformation U_{2m} on molecules 2 and m with $\theta = \pi/2^{m-3}, \cdots$, and the transformation $U_{m-1,m}$ on molecules $m-1$ and m with $\theta = \pi/4$ as in step (2). Finally, we perform a Hadamard transformation on quantum dot molecule m. The state of quantum dot molecule m becomes $|\varphi_1\rangle$.

(III) We detect the quantum dot molecules 1, 2, $\cdots$, m using detectors, and read out the result in reverse order. The measurement result is $|\varphi_m\varphi_{m-1}\cdots\varphi_1\rangle$, but the readout is $|\varphi_1\varphi_2\cdots\varphi_m\rangle$, so the estimated phase is $\widetilde{\phi} = \phi = 2\pi 0.\varphi_1\cdots\varphi_m$, which is precise.

2.3. Remarks on phase estimation and calibration of the timepiece

In the above process, we have assumed that φ can be expressed exactly in $\kappa = m$ qubits, but it is only an ideal case. For an arbitrary value of φ, and $\kappa < m$, if we wish to approximate φ up to an accuracy of $1/2^n$, then the successful probability should be about $1 - 1/(2^{m-n+1} - 4)$ with $m \geqslant n + 1$. The unknown phase φ can be created by modulating the length $l = vT$ of an electro-optic crystal (such as a KDP crystal), so we can estimate the time T in terms of $\phi_1 = 2\pi\varphi + \pi/2 = \varpi n_0^3 v r_{63} ET/2c = \varpi n_0^2 n r_{63} ET/2$, where ϖ is the frequency of an electric field, r_{63} is an electro-optic tensor, v is the velocity of a laser through the electro-optic crystal, and n and n_0 are refractive rates for the vacuum and electro-optic crystal, respectively. In the process of implementing phase estimation, the time T is detected by clock 2; if clock 2 undergoes h scales of a total of O scales, we can calculate the total time from $T_{\text{total}} = OT/h$ around a circle in clock 2. In the ideal case, we compare the T_{total} with the time T_i, which is the total time around a circle in an ideal clock. If $T_{\text{total}} = T_i$, clock 2 is an accurate one; otherwise, the frequency of clock 2 should be regulated. In the general case, we should first determinate the error of the phase $\eta = \widetilde{\phi}/\phi \times 100\%$; then we can calculate the error $\eta' = T_{\text{total}}/T_i \times 100\%$ of clock 2. If $\eta' \leqslant \eta$, we can treat clock 2 as an accurate one; otherwise, the frequency of clock 2 has to be regulated. In the case of $T_{\text{total}} < T_i$, the frequency of clock 2 should be increased; otherwise, the frequency should be decreased. In summary, calibration of a timepiece includes two aspects: one is checking whether the clock 2 is precise or not; the other is that if the clock 2 is not precise, we will regulate the frequency of clock 2 according to the error of the estimated phase. Similarly, we also can estimate the length l of an electro-optic crystal on the basis of the procedure of quantum phase estimation.

3. Discussion and conclusions

We discuss the feasibility of the current scheme with experimental parameters reported in current experiments. For general weakly coupled double-quantum-dot molecules, the strength of coupling t between $|e\rangle$ and $|i\rangle$ is about 0.01 meV, and the energy difference $\Delta = E_e - E_i$ is about 10 meV; thus the spatial separation factor $\gamma = t^2/(\Delta^2 + t^2) \simeq 10^{-6}$ [12]. In our scheme, we use two laser pulses with different coupling strengths Ω_1, Ω_2, which will satisfy the condition $\Omega_1 \sim 10^{-3}\Omega_2$ according to the above value of γ. For the process of the interaction involving two photons, the strength of the coupling Ω_c caused by the cavity field is 300 MHz [13, 15], where we have assumed that $\Omega_2 = 0.1$ meV and $\delta = 1$ meV as done in [12], resulting in $\Omega_{\text{eff}} = \Omega_2\Omega_c/\delta \simeq 30$ kHz and $\Omega_1 \simeq 10^{-4}$ meV. Therefore completion of a single-qubit operation and a two-qubit operation will cost about several hundred nanoseconds

and 10^{-4} s, respectively. We can calculate the total time for completing the current scheme, which is about $T = n(n-1)/2 \times 10^{-4}$ s. The coherent time of the spatial separate charge qubits can reach tens of seconds [18] (we can assume $T_c = 10$ s). Comparing the time T with T_c, it is shown that the number of qubits will be $n \simeq 450 \gg 100$ if $T = T_c$, so our scheme is suitable for large-scale quantum computation in quantum dot systems.

In conclusion, we present a scenario for implementing general quantum phase estimation via weakly coupled double-quantum-dot molecules embedded in a microcavity. The two quantum dots involved are not necessarily exactly identical, which reduces the experimental difficulty. In the same process of our implementation of quantum phase estimation, we can also realize the calibration of a timepiece or estimation of length. The key ingredient for our scheme is implementing the C–U transformation and the reversed QFT. In addition, the error of the time (length) can be calculated using the fidelity of the quantum phase. In other words, an arbitrary classical quantity related to the estimated quantum phase can be estimated by the same method. These classical estimation results (time T, length l, etc) are useful for our lives. The phase estimation would also be an important step for fabricating a quantum computer since it is the key ingredient for complex quantum algorithms. It also deserves note that our idea can be generalized to other systems, such as atom systems, trapped ion systems and linear optic systems.

Acknowledgments

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